PROJECTED WRITTEN NOTES FROM THE M325K LECTURE ON THURSDAY, JANUARY 25, 2024 on

ARGUMENTS IN LOGIC AND PREDICATE CALCULUS

CLASS #14

Another Connective: IFF = "If AND ONLY IF" — $\underbrace{\text{If AND ONLY IF}}_{\text{The Biconditional}}$

The Biconditional: "p if and only if q", "p IFF q"

"$p \leftrightarrow q$"

The Assertion: p and q are both True or are both False.

"p, IF q" AND "p ONLY IF q"

p	q	p IFF q $p \leftrightarrow q$	$(q \to p)$	$\wedge$	$(p \to q)$
T	T	T	T	T	T
T	F	F	T	F	F
F	T	F	F	F	T
F	F	T	T	T	T

$\equiv$

Ex: Assume that n is an integer.

Let p = "$\cos(n\pi) = -1$". Let q = "n is odd."

$p \leftrightarrow q$ = "$\cos(n\pi) = -1$ if and only if n is odd."

The Biconditional is True here.

Def'n: An argument (argument form) is a list of statements (statement forms) and the last one in the list is called the Conclusion (and it begins with the word "Therefore" ($\therefore$))

The previous statements (statement forms) are called Premises.

A valid Argument (argument form) is one such that, whenever all the premises are TRUE, the conclusion is also True.

An Argument (or argument form) is Invalid (and is called a "fallacy") if there is at least one case in which all the premises are true, but the conclusion is FALSE.

An Argument Form Ex:

$$\begin{array}{l} p \to q \\ p \\ \hline \therefore q \end{array}$$

The standard Valid Argument form called Modus Ponens.

An Argument: (let x be a real #)

If $x > 1$, then $x > 0$.
$x > 1$

$\therefore x > 0$

This argument is Valid.

PROVING "MODUS PONENS" IS VALID.

p	q	$p \to q$	p	q
		The Premises		Conclusion
T	T	T	T	T
T	F	F	T	F
F	T	T	F	T
F	F	T	F	F

T!!

← These are Critical Row

This argument is a Valid ~~Argument~~.

All the Critical Rows have the Conclusion True.

Consider the argument form:

$$\begin{array}{l} p \to (q \vee \sim r) \\ q \to (p \wedge r) \\ \hline \therefore\, p \to r \end{array}$$

The Truth TABLE Analyzing this argument is on p. 51.

Testing an Argument Form for Validity

1. Identify the premises and conclusion of the argument form.
2. Construct a truth table showing the truth values of all the premises and the conclusion.
3. A row of the truth table in which all the premises are true is called a **critical row**. If there is a critical row in which the conclusion is false, then it is possible for an argument of the given form to have true premises and a false conclusion, and so the argument form is invalid. If the conclusion in *every* critical row is true, then the argument form is valid.

Example 2.3.1 Determining Validity or Invalidity

Determine whether the following argument form is valid or invalid by drawing a truth table, indicating which columns represent the premises and which represent the conclusion, and annotating the table with a sentence of explanation. When you fill in the table, you only need to indicate the truth values for the conclusion in the rows where all the premises are true (the critical rows) because the truth values of the conclusion in the other rows are irrelevant to the validity or invalidity of the argument.

$$p \to q \vee \sim r$$
$$q \to p \wedge r$$
$$\therefore\ p \to r$$

Solution The truth table shows that even though there are several situations in which the premises and the conclusion are all true (rows 1, 7, and 8), there is one situation (row 4) where the premises are true and the conclusion is false.

						premises		conclusion
p	q	r	$\sim r$	$q \vee \sim r$	$p \wedge r$	$p \to q \vee \sim r$	$q \to p \wedge r$	$p \to r$
T	T	T	F	T	T	T	T	T
T	T	F	T	T	F	T	F	
T	F	T	F	F	T	F	T	
T	F	F	T	T	F	T	T	F
F	T	T	F	T	F	T	F	
F	T	F	T	T	F	T	F	
F	F	T	F	F	F	T	T	T
F	F	F	T	T	F	T	T	T

This row shows that an argument of this form can have true premises and a false conclusion. Hence this form of argument is invalid.

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Modus Ponens and Modus Tollens

An argument form consisting of two premises and a conclusion is called a **syllogism.** The first and second premises are called the **major premise** and **minor premise**, respectively.

Standard Valid Argument Forms

Modus Ponens

If p, then q.
p

$$\therefore q$$

Modus Tollens

If p, then q.
$\sim q$

$$\therefore \sim p$$

Generalization

$$\frac{p}{\therefore p \vee q} \qquad \frac{q}{\therefore p \vee q}$$

Specialization

$$\frac{p \wedge q}{\therefore p} \qquad \frac{p \wedge q}{\therefore q}$$

Conjunction

$$\begin{array}{l} p \\ q \\ \hline \therefore p \wedge q \end{array}$$

Elimination

$$\begin{array}{l} p \vee q \\ \sim p \\ \hline \therefore q \end{array} \qquad \begin{array}{l} p \vee q \\ \sim q \\ \hline \therefore p \end{array}$$

Transitivity of Implication

$$\begin{array}{l} p \to q \\ q \to r \\ \hline \therefore p \to r \end{array}$$

Proof by Division into Cases

$$\begin{array}{l} p \vee q \\ p \to r \\ q \to r \\ \hline \therefore r \end{array}$$

(Proof-by-) Contradiction Rule

$$\frac{\sim p \to (r \wedge \sim r)}{\therefore p}$$

Proof by Contradiction Rule:

If one can show that the hypothetical assumption that p is False leads logically to a contradiction, then one can conclude that p is True.

Standard Fallacy Forms

Fallacy of the Converse (Converse Error)	Fallacy of the Inverse (Inverse Error)
If p, then q. q ――――― $\therefore$ p	If p, then q. ~p ――――― $\therefore$ ~q

The Fallacy of the Converse gets its name from the fact that it comes about from an attempt to apply Modus Ponens, but confusing the conditional with its converse.

The Fallacy of the Inverse gets its name from the fact that it comes about from an attempt to apply Modus Ponens, but confusing the conditional with its inverse.

Predicate Calculus

Definition: A Predicate (Open Sentence) is a sentence that contains one or more variables, and the sentence becomes a logical statement when values are substituted for the variables.

Examples: P(x): "x^2 is positive."
Q(x,y): "x is the daughter of y"

P(x) is true when x represents 3.
P(x) is false when x represents 0.

The Domain of a variable is the set of values that can replace that variable, i.e., the values that the variable can represent.

The Truth Set of predicate P(x) is the set of all values in the domain of x for which P(x) is true when x is replaced by those values. The Truth Set = $\{ x \in \text{Domain } D \mid P(x) \text{ is true} \}$

A Quantifier is a phrase added to a predicate which makes an assertion concerning the number of values in the domain of the variable that make the predicate true.

The Universal Quantifier ($\forall$) asserts that every value in domain makes the attached predicate true.

Phrases: "For all $x \in D$, P(x)." ; "For every $x \in D$, P(x)." ;
"For any $x \in D$, P(x)." In Symbols: $\forall x \in D, P(x)$.

These statements are called Universal Statements.

An example of the Truth Set of a Predicate:

Let n be a positive integer. Then, $n \in \mathbb{Z}^+$

Predicate $P(n)$ is "$n \le 5$".

The Truth Set of $P(n)$ is $\{1, 2, 3, 4, 5\}$

$= \{n \in \mathbb{Z}^+ \mid n \le 5\}$

Universally Quantified Statements ("Universal Statements")

The Quantifier: "For every even integer n", ...

A formal wording has a variable: "For every even integer n, n^2 is even."

Informal wordings: "Every even integer has an even square"

"Any even integer " " " " "

In Symbols: $\forall n \in \mathbb{Z}^{EVEN}, n^2 \in \mathbb{Z}^{EVEN}$.

T or F? "For all real numbers t, $t^2 > 0$."

$\forall t \in \mathbb{R}, t^2 > 0$.

$0^2 = 0 \not> 0$

$0^2 \le 0$

$t = 0$ is a counterexample.
The universal statement is F.

Existential Statements

The Existential Quantifier

The predicate

These are the only two ways to word an Existential Quantifier

There exists a negative integer n such that $n^2=49$.

(True, $n=-7$ works).

$n^2=49$ for some negative integer n.

In Symbols: $\exists n \in \mathbb{Z}^{NEG}$ such that $n^2=49$

There is a negative integer whose square is 49.

When a variable is defined in a Universal stmt (For all...), the variable loses its definition outside of the statement

When a variable is defined in an existential statement, the variable is defined globally:

"There exists an integer n such that $n^2=81$."

$\equiv$ There exists an integer whose square is 81.

Let n be such an integer.

The Existential Quantifier ($\exists$) asserts that at least one value in domain makes the attached predicate true.

Phrases: "There exists a value $x \in D$, such that $P(x)$." ;
"For at least one $x \in D$, $P(x)$." ;
"For some $x \in D$, $P(x)$." ; "There is a value for x ..."
In Symbols: $\exists x \in D$ such that $P(x)$.

These statements are called Existential Statements.

Truth Values of Quantified Statements

The universal statement with form "$\forall x \in D, P(x)$" is defined to be True if, and only if, $P(x)$ is true for every value for x in the domain D. It is False when there is at lease one value for x in D such that $P(x)$ is false for that value, and any particular value of x which makes $P(x)$ false is called a counterexample of the universal statement.

The existential statement with form "$\exists x \in D$ such that $P(x)$" is defined to be True if, and only if, $P(x)$ is true for at least one value for x in the domain D. It is False when $P(x)$ is false for every value for x in D.

The declaration and definition of a variable made within a universal statement is applied locally and only within that universal statement.

The declaration and definition of a variable made within an existential statement is applied globally, except when used as the predicate of a universal statement.